density of heat carrier, kg/m^3; w, mass velocity of flow in the holes of the perforation, $kg/(m^2 \cdot sec)$; ρ_{pl}, porosity of the perforated plate; $Re = wd_0/\mu$, Reynolds number; d_0, diameter of hole of perforation, m; μ, absolute viscosity of heat carrier, $N \cdot sec/m^2$; $St = \alpha/(wc_p)$, Stanton number; α, heat-transfer coefficient, $W/(m^2 \cdot {}^\circ K)$; c_p, specific heat of heat carrier, $J/(kg \cdot {}^\circ K)$; $Pr = \mu c_p/\lambda$, Prandtl number; λ, thermal conductivity of heat carrier, $W/(m \cdot {}^\circ K)$; C, n, empirical coefficients.

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HEAT AND MASS TRANSFER IN A SUBMERGED AXISYMMETRIC TWISTED JET

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UDC 532.517;532.526

The excess-temperature distribution in a twisted flow is obtained for numbers $Pr \neq 1$, and the distribution of the concentration of a gaseous impurity is investigated.

L. G. Loitsyanskii was the first to formulate and analytically solve the problem of the development of a laminar submerged axisymmetric twisted jet of a viscous incompressible fluid [1]. The problem was solved on the basis of the equations of a laminar boundary layer by the method of asymptotic expansions. Loitsyanskii found the first and second terms of the expansions of the component velocities in their final form, these velocities corresponding to slightly twisted jets. For jets with a moderate twist, characterized by a "dip" of the longitudinal velocity on the jet axis, the authors of [2, 3] found the succeeding terms of the velocity and pressure expansions. In [4], heat exchange in an axisymmetric nonsimilitudinous jet at Pr = 1 was examined. In [5], the distribution of excess temperatures in submerged axisymmetric jets was found for arbitrary Prandtl numbers, and results of experimental studies were presented.

The present work is a logical continuation of [5]. The excess-temperature distribution is obtained for twisted jets for a wide range of Prandtl numbers. Results are presented from experimental studies of the distribution of a gaseous impurity in axisymmetric turbulent twisted jets of air. These results are compared with the solution obtained.

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Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 45, No. 1, pp. 36-42, July, 1983. Original article submitted April 19, 1982.

1. Laminar Jet. The equation of heat transfer in a laminar boundary layer of a viscous incompressible fluid in the case of axisymmetric motion in a cylindrical coordinate system has the form

$$u\frac{\partial \Delta T}{\partial x}+v\frac{\partial \Delta T}{\partial r}=a\left(\frac{\partial^2\Delta T}{\partial r^2}+\frac{1}{r}\frac{\partial \Delta T}{\partial r}\right). \quad (1)$$

An integral variant of the problem, besides the conditions of conservation of momentum K_0 and moment of momentum L_0 [1], is the condition of conservation of the excess heat content of the twisted jet [6]

$$Q_0=2\pi\rho c_p\int_0^\infty u\Delta T r dr=\text{const.} \quad (2)$$

We introduce the new independent variables [1]

$$X=x,\ \eta=r(x\nu)^{-1} \quad (3)$$

and to the asymptotic expansions of the stream function, excess pressure, and twist velocity w obtained by solving the dynamic problem

$$\Psi=\nu\left(\bar{a}X+a_0+\frac{a_1}{X}+\frac{a_2}{X^2}+\ldots\right),$$

$$\frac{P}{\rho}=\frac{c_1}{X}+\frac{c_2}{X^2}+\frac{c_3}{X^3}+\ldots,\quad w=\frac{b_1}{X}+\frac{b_2}{X^2}+\frac{b_3}{X^3}+\ldots,$$

we add the expansion of excess temperature

$$\Delta T=\frac{d_1}{X}+\frac{d_2}{X^2}+\frac{d_3}{X^3}+\ldots \quad (4)$$

Here $\bar{a}$, a_0, $a_1,\ldots$, c_i, b_i ($i = 1, 2\ldots$) are functions determined in the above-noted works; d_i are unknown functions of η. Insertion of the expansions into (1) and comparison of the coefficients of the terms with identical powers of X gives us the following system of ordinary differential equations for determining d_1, d_2, $d_3\ldots$:

$$d_1''+\frac{1+\Pr\bar{a}}{\eta}d_1'+\Pr\frac{\bar{a}'}{\eta}d_1=0,$$

$$d_2''+\frac{1+\Pr\bar{a}}{\eta}d_2'+2\Pr\frac{\bar{a}'}{\eta}d_2=-\Pr\frac{a_0'}{\eta}d_1,$$

$$d_3''+\frac{1+\Pr\bar{a}}{\eta}d_3'+3\Pr\frac{\bar{a}'}{\eta}d_3=-2\Pr\frac{a_0'}{\eta}d_2-$$

$$-\Pr\frac{a_1'}{\eta}d_1+\Pr\frac{a_1}{\eta}d_1', \quad (5)$$

. . . .

Similarly, from (2) we obtain the system of integral conditions

$$\int_0^\infty \bar{a}d_1 d\eta=\frac{Q_0}{2\pi\mu c_p},\quad \int_0^\infty(\bar{a}'d_2+a_0'd_1)\,d\eta=0,$$

$$\int_0^\infty(\bar{a}'d_3+a_0'd_2+a_1'd_1)\,d\eta=0,\ \ldots \quad (6)$$

In accordance with the boundary conditions in [6]

$$d_i(0)<M\ (\text{bounded}),\quad d_i(\infty)=0\ (i=1,2,\ldots) \quad (7)$$

In the new variable ζ [1]:

$$\zeta = \frac{1}{4}\alpha^2\eta^2\left(1 + \frac{1}{4}\alpha^2\eta^2\right)^{-1}, \quad \alpha = \sqrt{\frac{3K_0}{16\pi\mu}} \tag{8}$$

system (5) reduces to the hypergeometric equations

$$\zeta(1-\zeta)d_1'' + [1 - 2(1-\Pr)\zeta]d_1' + 2\Pr d_1 = 0,$$

$$\zeta(1-\zeta)d_2'' + [1 - 2(1-\Pr)\zeta]d_2' + 4\Pr d_2 = \beta\bar{\alpha}^2\Pr(1-4\zeta)(1-\zeta)^{2\Pr},$$

$$\zeta(1-\zeta)d_3'' + [1 - 2(1-\Pr)\zeta]d_3' + 6\Pr d_3 = -\frac{\beta^2\bar{\alpha}^2}{2}\Pr\left[\frac{3}{2} - (3\Pr+9)\zeta + (12\Pr+6)\zeta^2\right](1-\zeta)^{2\Pr} -$$

$$-\frac{\gamma^2\bar{\alpha}^2}{3\alpha^4}\Pr\left(5 + (10\Pr - 19)\zeta - 14(\Pr-1)\zeta^2\right)(1-\zeta)^{2\Pr-1}. \tag{9}$$

The solution of the first equation of system (9) satisfying boundary conditions (7) and the first integral condition (6) has the form [1]

$$d_1(\zeta) = \bar{\alpha}^2 F(-2\Pr, 1, 1, \zeta) = 2\bar{\alpha}^2(1-\zeta)^{2\Pr}. \tag{10}$$

Here

$$\bar{\alpha} = \sqrt{\frac{Q_0(1+2\Pr)}{16\pi\mu c_p}}. \tag{11}$$

The solution of the second equation of system (5) satisfying boundary conditions (7) and the second integral condition (6) was obtained in [5]:

$$d_2(\zeta) = -\frac{\beta\bar{\alpha}^2}{2}(1 - 4\Pr\zeta)(1-\zeta)^{2\Pr}, \tag{12}$$

where $\beta = M_0/2\pi\mu$, while M_0 is the second mass flow rate of the liquid through the jet cross section [1].

Depending on the value of the Prandtl number, there is a homogeneous equation in the neighborhood of the point $\zeta = 0$ — being a branch point of the solutions of the hypergeometric equations — which corresponds to the third equation of system (5). It has a logarithmic singularity at $\Pr \neq 1$ and has no singularities at $\Pr = 1$ [7]. In the right side of the equation for d_3, the term with γ^2 reflects the effect of the rate of twist on the temperature distribution in the jet.

The solution for d_3 at $\Pr = 1$ was obtained in [4]. The general solution of the equation for d_3 at $\Pr \neq 1$, satisfying boundary conditions (7) and the third integral condition (6), has the form

$$d_3(\zeta) = \left\{\frac{\beta^2\bar{\alpha}^2}{8}\left(1 - 10\Pr\zeta + 4\Pr(2\Pr+1)\zeta^2\right) -\right.$$

$$\left. -\gamma^2\frac{\bar{\alpha}^2}{3\alpha^4(\Pr-1)}\left(1 + \Pr(5\Pr-7)\zeta + \Pr(\Pr-1)\sum_{n=2}^{\infty}\frac{a_n}{n!}\zeta^n\right)\right\}(1-\zeta)^{2\Pr},$$

$$a_2 = 0, \quad \frac{a_n}{n!} = -\frac{4\Pr}{n^2} + \frac{a_{n-1}}{(n-1)!}\left(\frac{n-1}{n} + 2\Pr\frac{n-3}{n^2}\right),$$

$$(n \geqslant 3) \quad \gamma = \frac{3\sqrt{3}}{64\pi\sqrt{\pi}}\frac{L_0\sqrt{\rho K_0}}{\mu^2}. \tag{13}$$

The expansion of (4) in the variable η, in accordance with (8), is written

$$\Delta T = \frac{2\bar{\alpha}^2}{\left(1 + \frac{1}{4}\alpha^2\eta^2\right)^{2\Pr}}\left[\frac{1}{X} - \frac{\beta}{4}\frac{1 + (1-4\Pr)\frac{\alpha^2\eta^2}{4}}{\left(1 + \frac{1}{4}\alpha^2\eta^2\right)}\frac{1}{X^2} + \right.$$

$$+\left(\frac{\beta^2}{16}\frac{1 + \frac{1}{4}\alpha^2\eta^2(2-10\Pr) + \frac{\alpha^4\eta^4}{16}(1 - 6\Pr + 8\Pr^2)}{\left(1 + \frac{1}{4}\alpha^2\eta^2\right)^2} - \right.$$

TABLE 1. Integral and Characteristic Constants of Twisted Jets

Type of jet	Ω	K_0, N	$L_0 \cdot 10^2$, N·m	$\alpha/\sqrt{d}$	β/d	γ/d^2	δ/d^3	$\bar{\alpha}$	k_η
C-1	1,86	1,16	0,553	11,72	10,20	54,77	559	0,140	0,67
C-2	2,08	1,06	0,552	11,62	8,89	235,26	575	0,132	0,76
C-3	2,48	1,30	0,804	11,40	6,16	609,92	1059	0,116	0.65

$$-\frac{\gamma^2}{3(\mathrm{Pr}-1)\alpha^4}\left(\frac{1+\frac{1}{4}\alpha^2\eta^2(1-7\mathrm{Pr}+5\mathrm{Pr}^2)}{1+\frac{1}{4}\alpha^2\eta^2}+\mathrm{Pr}(\mathrm{Pr}-1)\sum_{n=2}^{\infty}\frac{a_n}{n!}\left(\frac{\frac{1}{4}\alpha^2\eta^2}{1+\frac{1}{4}\alpha^2\eta^2}\right)^n\right)\Bigg)\frac{1}{X^3}\Bigg]. \tag{14}$$

2. Turbulent Jet. Following the hypothesis of L. G. Loitsyanskii [1], we will examine an axisymmetric turbulent twisted jet as a laminar jet but with molar viscosity. It should be noted that this assumption will be less valid in regions adjacent to zones of reverse flows (in the zone of the "dip" of longitudinal velocity near the jet axis), where the corresponding turbulent motion will be quite complex. However, as the results of experimental studies [3, 8, 9, 10] show, such a simplified picture of turbulent motion gives a qualitatively correct picture of flow in the jet (see [11] also). Thus, the results obtained for a laminar jet are assumed to hold true for a turbulent jet on the assumption that the velocity is averaged over time and that we can replace the molecular viscosity μ and kinematic viscosity ν by the molar viscosity A and turbulent kinematic viscosity ε. In accordance with this, the Prandtl number of a laminar flow is replaced by the turbulent Prandtl number Pr_t. Since the mass-transport process in jets is equivalent to the heat-transfer process [6, 8], these processes are described by the same equations, with a corresponding substitution of the expression of concentration c for the temperature ΔT and the replacement of the thermometric conductivity by the diffusion coefficient, i.e., the replacement of Pr_t by Sc. Thus, the distribution of the concentration of an impurity in turbulent submerged jets is described by Eq. (14) with allowance for the above considerations. According to [8], the number Sc = 0.7. The velocity and pressure distributions in the jet are given in [2, 3].

We conducted experimental studies of the distribution of the concentration of a gaseous impurity in an axisymmetric turbulent twisted jet of air issuing from a nozzle with Ø 10.5 mm (flow rate 24 m^3/h, Re = $5.6 \cdot 10^4$). We added the gaseous impurity to the pipe supplying the nozzle with air. The impurity was methane, and the amount added was 5% of the air flow rate through the nozzle. Twisted jets were formed by means of four-pass screw-type swirlers installed in succession in the body of the nozzle. Velocities and pressure in the jets were measured with a ball-type five-channel probe (diameter of head 4 mm), which allowed us to obtain three velocity components and the distribution of static pressure. A Pitot–Prandtl tube was used to sample the gas. The recording devices were MMN-250 micromanometers and a "Tsvet" gas chromatograph with a KSP-4 recording potentiometer. The accuracy of measurement of the concentration of methane in the mixture was within 3%.

The criterion used to evaluate the intensity of twisting of the jet was the criterion [12] $\Omega = 4L_0/K_0 d$. Integral characteristics of the jets investigated are shown in Table 1.

The characteristic constants α, β, γ, δ, $\bar{\alpha}$ were determined by comparing the distributions of velocity, pressure, and concentration along the jet axis that were obtained with the theoretical distribution curves of these quantities with $\alpha\eta = 0$ [3]. The variable η is connected with the radius of the jet by the formula [3] $\alpha\eta = k_\eta r_0/\bar{X}$, where k_η is a proportionality factor. Values of this factor are shown in Table 1.

Figure 1 shows the change in the maximum values of the concentrations c_m/c_0 along the jet axis $\bar{X} = X/d$. The dashed curves correspond to the similitudinous solution (the first term is entered into the concentration expansion (14) with $\alpha\eta = 0$), while the solid curves were obtained from Eq. (14). It is apparent from the figure that the boundary between the transitional and main (similitudinous) sections of the jet can be considered to be the section where curves 1 and 2 nearly merge. Meanwhile, this distance from the nozzle edge depends on the intensity of the twisting, and for the investigated jets C-1 $\bar{X}$ = 25, C-2 $\bar{X}$ = 30, and C-3 $\bar{X}$ = 34. Since the similitude boundary moves farther from the nozzle edge with an increase in Ω — which leads to a change in the boundaries of the transitional zone — the solution obtained

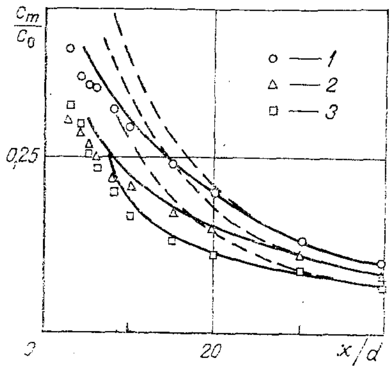

Fig. 1. Change in maximum concentration of gaseous impurity c_m/c_0 along the axis $\bar{X}$: 1) C-1; 2) C-2; 3) C-3. The dashed curves represent the similitudinous solution; the solid lines represent the nonsimilitudinous solution.

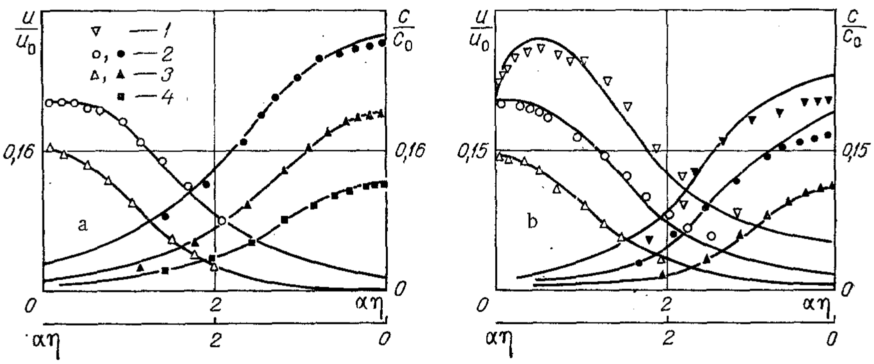

Fig. 2. Distribution of velocity u/u_0 and concentration c/c_0 at different sections of the jet: 1) X/d = 8; 2) 10; 3) 20; 4) 30; the clear points are the results of velocity measurements; the dark points are measured concentrations; a) C-1; b) C-3.

makes it possible to describe the concentration distribution up to 10 nozzle diameters for C-1 (the maximum difference between the theoretical and experimental values of concentration was 3%), up to 15 diameters for C-2 (5% maximum difference), and up to 20 diameters for C-3.

Figure 2 shows the results of calculations of the distribution of velocity u/u_0 (according to Eqs. (1) and (2)) and concentration c/c_0 (Eq. (14)) for jets C-1 and C-3 at sections X/d of the transitional and main sections of the jets. It is apparent from the figure that the jet region characterized by the "dip" in axial velocity is described well by the formulas for velocity, although a certain difference between the experimental and theoretical results is seen for concentration.

NOTATION

x, r, longitudinal and transverse coordinates; u, v, longitudinal and radial components of velocity; ΔT, excess (relative to the ambient temperature) temperature; a, thermometric conductivity; ν, kinematic viscosity; Q_0, heat flux; c_p, specific heat at constant pressure; Sc, Schmidt number; u_0, mean flow-rate velocity; c_0, initial concentration of methane impurity.

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EXPERIMENTAL STUDIES OF A PLANE BUOYANT SURFACE JET

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UDC 532.543

Results are presented from an experimental study of the conditions of formation of a plane buoyant jet with different initial values of the density Froude number.

The need to study buoyant surface jets crises in solving a number of important practical problems such as discharge into special cooler-reservoirs or reservoirs designed for multipurpose use of the heated water from heat and electric power plants, as well as the flow of fresh water into bodies of salt water (estuaries, seas, lakes).

The main feature of fluid flow being examined here is the effect of buoyancy on the propagation of the jet under conditions of stable stratification. The presence of stable stratification creates boundaries between layers of liquid of different densities which are stable with respect to time and space.

The most serious shortcoming of earlier studies of plane buoyant surface jets [1-5] is the absence of steady flow conditions during the experiments, which was due mainly to two factors: the flow of heavy liquid from the working volume via a spillway, and the appearance of a water whirlpool region.

To neutralize the effect of the first factor, the authors of [3] arranged for forced delivery of heavy liquid into the bottom part of the working compartment at a rate exceeding the flow rate of the lighter liquid. This also introduced some distortion into the actual flow pattern, affecting particularly the mixing and entrainment parameters.

A problem involving artificial make-up is complicated, since as much heavy liquid needs to be added to the working compartment as is carried off by the flow of light liquid in the given flow regime. The amount of make-up water can be determined from the relation

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Institute of Hydromechanics, Academy of Sciences of the Ukrainian SSR, Kiev. Translated from Inzhnerno-Fizicheskii Zhurnal, Vol. 45, No. 1, pp. 42-50, July, 1983. Original article submitted March 12, 1982.

 0022-0841/83/4501-0742$07.50